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# The Graph Theory Application in the Praxis of Flight Path Planning

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**Abstract**

The article is focused on the graph theory application in the praxis of aviation - especially the flight routes planning and airways optimization. The article analyses the theoretical-practical findings of the graph matrix and consequently examines how to applicate into air transportation. The main aim of the article is to highlight the possible practical methods of using the graph and matrix theory in the optimizing of air tracks and routes and minimizing errors during tactical flight planning. The result of the exploration is an overview and response to the issue of graph theory application in aviation. The graph and matrix theory helps to ensure the effective, safe, economic and ecological air traffic flow management, optimize the tactical flight planning, minimize the unexpected situations and prevent the possible air traffic congestions and delays.
The result of the article demonstrates that graph and matrix theory is one of the possible solutions in the air traffic flow management and tactical flight planning issue.
**KEY WORDS:** *air traffic flow management, CPM method, flight path planning, flight routes, graph theory.*

## 1. Introduction

Graph theory is a mathematical field, and it is a special part of the combinatorial analysis, which closely relates to applied mathematics and operational analysis. It deals with the study of mathematical form - graph. The term of the graph can be defined from a mathematical and, from a normal point of view, in two ways:

1. graph as a way of displaying (for example, some dependencies of variables);
2. graph as a certain mathematical form, which is a model of some real system.

The importance of the graph theory is best described by the second definition.

The graph can be characterised as a form, showing relationships between elements of a system through a set of vertices and edges. The roots of the graph theory date back to the 18th century, when in 1736 the Swiss mathematician Leonard Euler solved the oldest and best-known graph issue, the Königsberg bridge problem. This already solved mathematical problem is based on a real place and a real situation. The German city (now it is Russian Kaliningrad) was situated on the river Pregel, which created two islands. The islands were connected with the other cities by seven bridges. The question was whether it was possible to take a walk through the island in such a way as to cross over every bridge only once. It was Euler who first solved the issue and stated that it is not possible. He reworded the problem based on his graph theory method and proved that in the created graph, there is no Eulerian move.

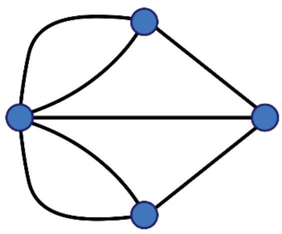

Fig. 1 The Königsberg bridge issue [2]

Only Eulerian graphs have one-stroke draw property. This above-mentioned issue is referred to as the beginning of graph theory. In summary, the graph optimization algorithms based on the graph theory is a very useful technique of transport system theory, which uses his own access and procedures for solving optimization problems.

The main goal of these optimization tasks is to search for a minimal graph frame, search for optimal routes and ways in the graph (minimal, maximal, shortest, longest, cheapest, etc. track or airways), determine sequences and maximal flows in tracks, design new routes, tracks and other. The graph theory is also studied and discussed in works [1, 3].

The application of acquired knowledge is part of the aviation training of pilots and air traffic controllers, as well as the flight planning dispatchers, which has a 65-year tradition in Košice, in the Slovak Republic. The core elements of education and training of pilots as a part of the Knowledge Alliance of Aviation Education are:

– the know-how, the past and present Civil and Military aviation maintenance, repair and operation's experience;

– the results of the theoretical work and the scientific and research activities in the field of: University Academic subjects, Social and Human Sciences, the Simulation and modelling of Security issues, the Applied Technical Sciences, the Applied Civil and Military / Air Force management, education and training etc. within the Expert Database of Civil and Military Aviation Experience in progess (the selected theoretical framework of Knowledge Alliance within the Simulation and modelling of Security issues is in the works of authors [4 - 8], the selected theoretical framework of Knowledge Alliance within the Applied Technical Sciences is in the next works of authors [9 - 12], the another selected theoretical framework of Knowledge Alliance within the Applied Civil and Military / Air Force management, education and training is in the works of authors [13]).

## 2. Basic Knowledge and Concepts in Graph Theory Important for Graph Travel

Each graph consists of certain parts that it is need to know for proper graph handling. The basic elements of graphs are a set of nodes or vertices and edges. It means that a graph is a collection of vertices and edges.

**Vertices** are elements of the set U and are places, where at least one of the following occurs:

– elements enter or exit the system, or gathers;
– it creates, cancels or manipulates ensembles.

**Edges** or lines are pairs of vertices in an undirected graph, arranged pairs of vertices in an oriented graph and a set of vertices in Multigraph. We can mark them as "$h_{ij}$" or as a disordered pair $[u_i, u_j]$. They are elements of the set $U$ (Fig. 2).

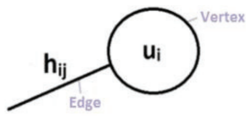


Fig. 2 Vertex-edge [own source]

The **orientated edge** is given by the starting and ending vertex, indicated by an arrow in the graph. It is an ordered pair $(u_i, u_j)$.

**Parallels edges** are at least two edges, which connect the same pair of vertices.

**Multiple edges** are at least two parallel edges that start and end at the same vertices. They differ from parallel ones in that they are all oriented or non-oriented (Fig. 3).

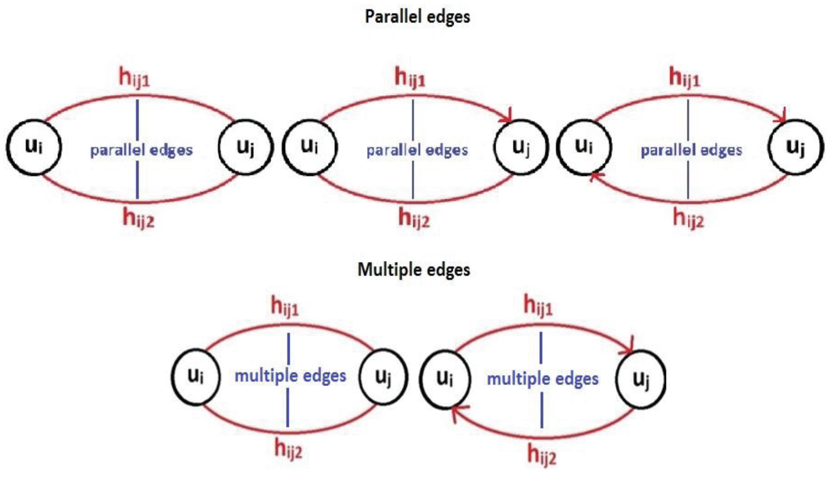


Fig. 3 Parallel and multiple edges [own source]

**Adjacent edges** are two edges that have a common vertex.

**Adjacent vertices** are two vertices, between which is an edge that connects them.

**A loop** is an edge that starts and ends in the same vertex. It is incidents with one vertex (Fig. 4).

**The sequence** is an alternating sequence of consecutive vertices and edges that starts and ends at a vertex. There are two types of sequences: open and closed.

*An open sequence* is a sequence in which the initial and final vertex is different, $u_1 \neq u_n$..

*A closed sequence* is a sequence in which the initial and final vertex is identical, $u_1 = u_n$.. It means, that a closed path is a cycle.

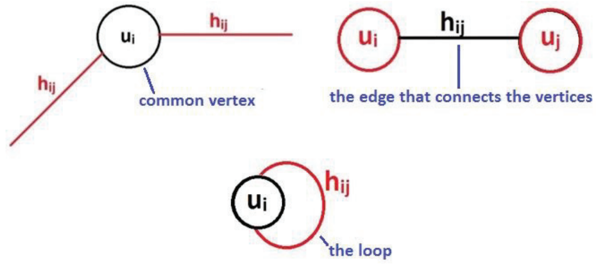


Fig. 4 Adjacent edges and vertices, and loop [own source]

**A cycle** is a closed path and has at least one edge. No vertex is repeated except the initial one which is at the same time final. It's a loop.

**A stroke** is a sequence in which no edge is repeated.

**A path** is a walk with no repeated vertices.

**A relation** is a relationship of two graph elements with the same dimension (edge-edge, vertex-vertex).

**An incident** is a relationship between two elements of different dimension (edge-vertex). Vertex $u_i \in U$ is incident with the edge $h_{ij} \in H$ of the graph $G = [U, H]$ if the edge $h_{ij}$ strats or ends in the vertex $u_i$ [1-3].

### 3. The Uses of the Graph Theory in the Praxis of Aviation

The practical application of graph theory in transport can have the following tasks divided into two main groups:

– designing transport networks and selecting subnetworks;
– tasks of traffic flow.

The first group means the construction of completely new, non-existent network and the selection of a certain subnetwork that meets certain criteria. In the second category, it is mainly the selection of the optimal track, path in an existing network. The path optimisation means the selection such track, which corresponds to the expected parameters when we use a track - the cheapest, shortest, fastest route. We look for optimal routes in the network in order to minimize the cost of making a trip (referred to as minimum route distances). Similarly, as the graph theory, the operational and system analysation deals with the solution of track optimisation in transport.

At searching for optimal routes is very important to consider that the track must be as reliable, short as possible and consequently the cheapest and most economical. At creating new routes, it is considering the maximal track capacity too. The optimal path searching is possible in two main methods:

– graph theory methods;
– CPM method.

The next example describes the optimal flight route (one way track) searching based on graph theory.

The vertices represent airports or waypoints, and the edges show flight routes. For a clearer calculation, the edges (flight routes) have recalculated nautical miles to simple values, which represent the track distance from point 1 (AD 1) to point 2 (AD 2).

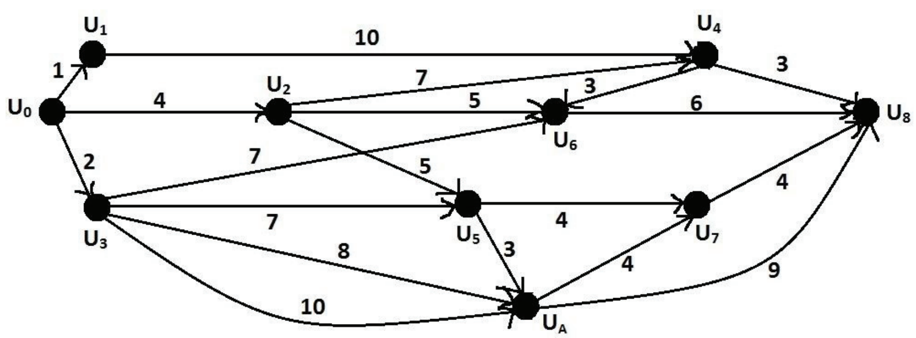


Fig. 5 Graph G – one way flight path example [own source]

From the theoretical point of view, the graph G is an oriented, direct and edge evaluated graph (Fig. 5).

With the help of graph theory, it is possible to determine the shortest way and describe in the following Table 1:

In the table, the shortest routes from the point $U_0$ to other points are marked in red. It means that from $U_0$ (start point) to $U_8$ (destination or final point) the shortest route has a value of 10. Table 2, in summary, describes the final shortest routes from point to point.

The shortest path can be calculated and graphically illustrated by the CPM method. The CPM method calculates the shortest distances from the start point (vertex) $U_0$ to the final point (vertex) $U_8$. $U_A$ in this example is the alternate or adjacent airport, point or vertex and $U_8$ is the final or destination point.

Unlike the CPM method used in the project management, where the longest path is calculated from the $U_i$ vertex to the $U_j$ vertex, in this case, it is the opposite - the shortest route.

Table 1

Shortest paths from point to point [own source]

| | $U_0$ | $U_1$ | $U_2$ | $U_3$ | $U_4$ | $U_5$ | $U_6$ | $U_7$ | $U_8$ | $U_A$ |
|---|---|---|---|---|---|---|---|---|---|---|
| D | 0 | 0+1 | 0+4 | 0+2 | 1+10<br>4+7 | 4+5<br>2+7 | 4+5<br>2+7<br>4+7+3<br>1+10+3 | 4+5+4<br>2+7+4<br>2+8+4<br>2+10+4 | 1+10+3<br>4+7+3<br>4+5+6<br>4+5+4+4<br>2+7+4+4<br>2+8+4+4<br>2+10+9 | 4+5+3<br>2+7+3<br>2+8<br>2+10 |
| D | 0 | 1 | 4 | 2 | 11<br>11 | 9<br>9 | 9<br>9<br>14<br>14 | 13<br>13<br>14<br>16 | 14<br>14<br>15<br>17<br>17<br>18<br>21 | 12<br>12<br>10<br>12 |

* the shortest paths from the point $U_0$

Table 2

Shortest paths from point to point [own source]

| | $U_0$ | $U_1$ | $U_2$ | $U_3$ | $U_4$ | $U_5$ | $U_6$ | $U_7$ | $U_8$ | $U_A$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $U_0$ | 0 | 1 | 4 | 2 | 11 | 9 | 9 | 13 | 14 | 10 |
| $U_1$ | - | 0 | - | - | 10 | - | 13 | - | 13 | - |
| $U_2$ | - | - | 0 | - | 7 | 5 | 5 | 9 | 10 | 8 |
| $U_3$ | - | - | - | 0 | - | 9 | 7 | 11 | 13 | 8 |
| $U_4$ | - | - | - | - | 0 | - | 3 | - | 3 | - |
| $U_5$ | - | - | - | - | - | 0 | - | 4 | 8 | 3 |
| $U_6$ | - | - | - | - | - | - | 0 | - | 6 | - |
| $U_7$ | - | - | - | - | - | - | - | 0 | 4 | - |
| $U_8$ | - | - | - | - | - | - | - | - | 0 | - |
| $U_9$ | - | - | - | - | - | - | - | 4 | 8 | 0 |

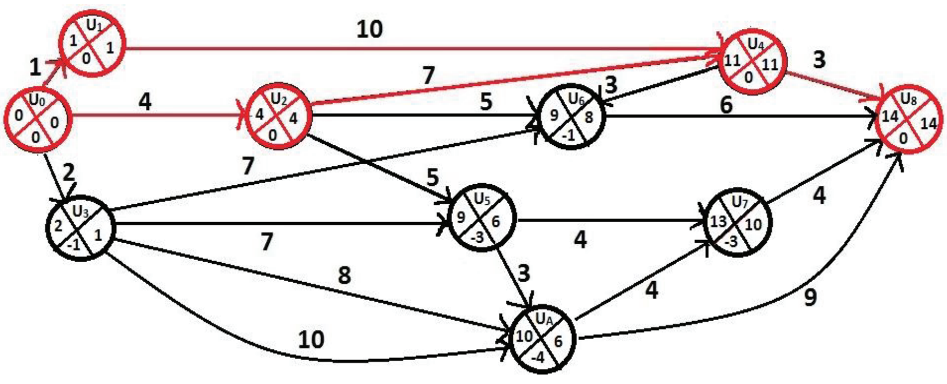

Fig. 6 Graph G - the shortest path by the CPM method [own source]

Fig. 6 illustrates the shortest track from the starting ($U_0$) to the final ($U_8$) vertex (point) in the graph G by a CPM method. The shortest routes are marked in red.

It is known from the above - mentioned graph that the flight path would extend:

- by 1 value through vertex $U_3$ and $U_6$;
- by 3 value through $U_5$ and $U_7$;
- by 4 value through $U_3$, $U_A$ and $U_7$.

In air transport are being constructed new air routes, or are being established optimal routes (shortest, most economical and ecological) for passenger, cargo and goods transport flight. In pre-flight briefing documentation, the pilots have available the favourable routes, generated by the scheduling system. The system works on the bases of graph theory, programming and algorithms. Determining the optimal route is a key factor in minimizing flight time, fuel consumption, emission, track length and delays. It ensures an organised and efficient air traffic flow management.

**4. Conclusions**

The safe, economical, ecological and fast air traffic flow is the key factor in aviation transport management. The main goal of the airline companies is to choose the optimal flight routes in terms of safety, distance, economy and length. Most of the air companies have a system for generating the optimal flight routes, which works on the bases of graph theory, system analysis and algorithms.

The methodology of transport system theory works with a set of concepts, methods and knowledge about transport, abstracting from the specific characteristics of individual modes of transport. As well as aviation, also a rail, road and ship transport department use the graph theory segment. The essence of the problem that transport systems theory solves in different transport departments is the same, and their mathematical model is similar. Therefore, it is important to choose the right method and then to correctly apply this method in the issue.

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